TA SESSION OF INTRODUCTION TO MATHEMATICAL ANALYSIS FOR GROUP 3

MARCH 1, 2022

(1) Consider the sequence of function $\mathcal{F} = \{f_n\}_{n\in\mathbb{N}} \subset C([0,1])$, which is defined by

$$f_n(t) = \sin(2^n t)$$

Note that the $C([0,1])$ is equipped with supremum norm.

(a) Show that $\mathcal{F}$ is bounded. Moreover, show that $\|f\| = 1$.

(b) Show that $\mathcal{F}$ is closed.

(c) Show that $\|f_n - f_m\| \geq 1$, for all $m \neq n$.

(d) $\mathcal{F}$ has no converge subsequence converge in $\mathcal{F}$.

(e) Explain why? Is $\mathcal{F}$ equicontinuous?

(f) [Extra] Is $\mathcal{F}$ totally bounded.

Remark: Recall Heine-Borel theorem. Given (X, d) a metric space and a set $E \subset X$, Then,

(a) E is compact in X.

(b) E is sequencially compact in X.

(c) E is closed and bounded in X.

(a) $\Leftrightarrow$ (b) $\Rightarrow$ (c). Could you rewrite (c) such that above are equivalent?