CALCULUS TA SESSION APRIL 26

(1) Fubini Theorem

Consider the function $f(x,y) = \frac{x^2-y^2}{(x^2+y^2)^2}$ for $(x,y) \in [0,1]\times[0,1]$ and compute the following integrals:

Hint: $\frac{\partial}{\partial x}\frac{-x}{x^2+y^2} = \frac{\partial}{\partial y}\frac{y}{x^2+y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$

(a) $u(y) = \int_0^1 f(x,y)\ dx$, if $y = 0$ and $0 < y \leq 1$

(b) $\int_0^1 \int_0^1 f(x,y)\ dxdy$

(c) $v(x) = \int_0^1 f(x,y)\ dy$, if $x = 0$ and $0 < x \leq 1$

(d) $\int_0^1 \int_0^1 f(x,y)\ dydx$

(e) $\iint_{[0,1]\times[0,1]} \max\{f(x,y), 0\}\ dA$

(f) $\iint_{[0,1]\times[0,1]} \max\{-f(x,y), 0\}\ dA$

(g) $\iint_{[0,1]\times[0,1]} |f(x,y)|\ dA$

(h) $\iint_{S_\epsilon} f(x,y)\ dA$, where $S_\epsilon = [0,1]\times[0,1] \setminus [0,\epsilon]\times[0,\epsilon]$

(i) $\iint_{[0,1]\times[0,1]} f(x,y)\ dA$

(j) **Why?** Does (a)(c) exist? Does (g) exist?