TA Session of Introduction to Mathematical Analysis

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1. Basic concept

   (a) In Euclidean space, is a continuos function map compact set to compact set?

   (b) In Euclidean space, is a continuos function map connected set to connected set?

   (c) In Euclidean space, does a set which is closed and bounded imply it is a compact set?

   (d) In metric, does a set which is closed and bounded imply it is a compact set?

   (e) In Euclidean space, is a continuous function map closed set to closed set?

   (f) In Euclidean space, is a continuous function map bounded set to bounded set?

   (g) In Euclidean space, is a continuous function map open set to open set?

   (h) In Euclidean space, is the inverse of a continuous function map open set to open set?

   (i) In metric space, is a connected set a path connected set?

   (j) In metric space is a complete set a compact set?

2. [Rudin] Problem 4.2 Let f and g be continuous mappings of a metric space X into a metric space Y, and let E be a dense subset of X. Prove that $f(E)$ is dense in $f(X)$. If $g(p) = f(p)$ for all $p \in E$, prove that $g(p) = f(p)$ for all $p \in X$. (In other words, a continuous mapping is determined by its values on a dense subset of its domain.)

3. [Rudin] Problem 4.4 If f is a continuous mapping of a metric space X into a metric space Y, prove that

$$f(E) \subset \overline{f(E)}$$

   for every set $E \subset X$. (E denotes the closure of E.) Show, by an example, that $f(E)$ can be a proper subset of $\overline{f(E)}$.