TA Session of Introduction to Mathematical Analysis

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1. Basic concept

   (a) In Euclidean space, is connected set imply path connected?

   (b) In Euclidean space, is continuous function imply uniformly continuous?

   (c) In Euclidean, is a function continuous at single point implies this function continuous on some neighborhood of this point?

   (d) In Euclidean space, is differentiable function imply continuously differentiable?

2. [Rudin] Problem 5.6 Suppose

   (a) f is continuous for $x \geq 0$,

   (b) $f'(x)$ exists for $x > 0$,

   (c) $f(0) = 0$

   (d) f' is monotonically increasing.

   Put

   $$g(x) = \frac{f(x)}{x} \quad (x > 0)$$

   and prove that g is monotonically increasing.

3. [Rudin] Problem 5.8 Suppose f' is continuous on $[a, b]$ and $\varepsilon > 0$. Prove that there exists $\delta > 0$ such that

   $$\left| \frac{f(t) - f(x)}{t - x} - f'(x) \right| < \varepsilon$$

   whenever $0 < |t - x| < \delta, a \leq x \leq b, a \leq t \leq b$.

4. 1051 A1 Midterm Problem 6 Suppose that f is a differentiable function. If $f'(a) > 0$ and $f'(b) < 0$, explain that there exists $c \in (a, b)$ such that $f'(c) = 0$