CALCULUS TA SESSION JANUARY 2 (VERSION 1)

TA: SINGYUAN YEH

(1) Definition *1012 A1 Midterm Problem 4*

Let

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases} .$$

Answer the following questions.

(i) Compute $f_x(0,0)$ and $f_y(0,0)$.

(ii) Compute $f_x(x,y)$ and $f_y(x,y)$ where $(x,y) \neq 0$.

(iii) Are $f_x(0,0)$ and $f_y(0,0)$ continuous at $(0,0)$?

(iv) Determine $f_{xy}(0,0)$ and $f_{yx}(0,0)$ if they exist. If they do not exist, explain why.

(v) Is f differentiable?

(2) Definition *1052 A1 Midterm Problem 4*

Consider the function

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+2y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(i) Find the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ or explain why the limit does not exist.

(ii) Compute the directional derivative $D_u f(0,0)$, where $u = (\cos\theta, \sin\theta)$ is any direction.

(3) Implicity Derivative *1042 A1 Midterm Problem 9*

Suppose that x, y, z satisfy the relation $x^2 + 2y^2 + 3z^2 + xy - z = 9$. Find $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y^2}$.

(4) Gradient *1072 A1 Midterm Problem 3*

Consider the level surface $z^2 + z\tan^{-1}\frac{y}{x} = \frac{\pi}{4} + 1$

(i) Find the tangent plane for the level surface at $(1,1,1)$.

(ii) The level surface defines z implicitly as a function of x and y, $z = f(x,y)$. Compute the directional derivative of $f(x,y)$ at $(1,1)$ in the direction of $(\frac{-3}{5}, \frac{4}{5})$.

(5) Gradient *1062 B Midterm Problem 5*

Find the tangent plane of $xe^{yz} + \ln(y^3 z^2) = \tan^{-1}\left(\frac{x}{z}\right)$ at $(0, 1, 1)$

(6) Directional Derivative *1052 A2 Midterm Problem 5*

Let $f(x, y, z) = (x^2 + z^2)\sin\left(\frac{\pi x y}{2}\right) + yz^2$ and a point $(1, 1, -1)$ called p.

(i) Find the gradient of f at p.

(ii) Find the approximate value of $f(0.98, 1.02, -0.97)$.

(iii) Find the plane tangent to the level surface through p defined by $f(p) = 3$

(iv) [*I dropped out this problem.*]

Note: Tell the difference between tangent to level surface (curve) and tangent to graph of function $z = f(x, y)$.

(7) Directional Derivative *1052 A1 Midterm Problem 5*

A differentiable function $f(x, y)$ has the following properties:

- $f(0, 0) = 1$
- $D_u f(0, 0) = 2$, where $u = (\frac{3}{5}, \frac{4}{5})$
- $D_v f(0, 0) = \frac{3}{\sqrt{2}}$, where $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Answer the following questions.

(i) What is the maximal rate of increase of $f(x, y)$ at $(0, 0)$?

(ii) Use the linearization of $f(x, y)$ at $(0, 0)$ to estimate $f(0.07, -0.05)$.

(8) Second Derivative Test *1062 A1 Midterm Problem 8*

Let $f(x, y) = 1 + 3x^2 - 2x^3 + 3y - y^3$

(i) Find the local maximum and minimum values and saddle point(s) of $f(x, y)$.

(ii) Find the extreme values of $f(x, y)$ on the region D bounded by the triangle with vertices $(-2, 2)$, $(2, 2)$ and $(2, -2)$.

(9) Lagrange Multiplier *1072 B Midterm Problem 4*

Find the points on the graph of the plane $x^2 + 4y^2 = 1$ such that $-x^2 + 4xy + 2y^2$ has maximum and minimum.

(10) (*) Lagrange Multiplier *1032 A1 Midterm Problem 11*

Find the points on the intersection of the plane $x+y+2z=2$ and the paraboloid $z=x^2+y^2$ that are nearest to and farthest from the origin.

(11) Double Integral *1052 A2 Final Problem 3*

Sketch the region of integration and evaluate the integral

$$\int_0^8 \int_{\sqrt{1+x}}^3 \cos(\frac{x}{y+1}) dydx\,.$$

(12) Coordinate Change *1032 A1 Final Problem 2*

Write the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z) dzdydx$$

in 5 other orders.

(13) Coordinate Change: Polar coordinate *1052 B Midterm Problem 7*

Evaluate

$$\iint_D \frac{1}{\sqrt{x^2+y^2}} dA\,,$$

where D is a region bounded by $r=1-\cos\theta$, $y=x$ and $y=-x$ with $y \geq 0$.

(14) Coordinate Change: Polar coordinate *1072 B Midterm Problem 6*

Evaluate

$$\iint_D x dA\,,$$

where D is a region bounded by $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq \sin 2\theta$.

(15) Coordinate Change: Polar coordinate *1032 A1 Final Problem 3(b)*

Evaluate

$$\int_1^{\sqrt{2}}\int_0^{\sqrt{2-y^2}} \frac{x+y}{x^2+y^2}dxdy + \int_0^1\int_{1-y}^1 \frac{x+y}{x^2+y^2}dxdy + \int_1^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} \frac{x+y}{x^2+y^2}dydx$$

(16) Coordinate Change: Jacobian *1062 B Midterm Problem 1*

Evaluate

$$\iint_\Omega (x-y)^{20}dA$$

where Ω is enclosed by $2x+y=0$, , $2x+y=1$, $x+2y=0$ and $x+2y=1$.

(17) Triple Integral *1052 A1 Final Problem 3*

Evaluate

$$\iiint_\Omega y\cos((y-z)^2)dV$$

where E is the solid tetrahedron bounded by four planes $x=1$, $y=1$, $z=0$ and $x+y-z=1$.