CALCULUS TA SESSION NOVEMBER 14 (VERSION 2)

(1) Volume of the revolution

Rotating $R = \{(x, y) : x \geq 1,\ 0 \leq y \leq \frac{1}{x}\}$ about the x-axis we obtain the solid. Find the volume of solid.

(2) Arc-length of the revolution

A catenary curve $y = \frac{e^x + e^{-x}}{2} = \cosh x$. Find the arc-length from $x = -100$ to $x = 100$.

(a) [*Extra*] Do you know the soup film?

(b) [*Extra*] Do you know the wormhole?

(3) Surface area of the revolution

Find the area fo the surface generated by revolving the curve $y = \cos x$, $x \in [0, \pi/2]$, about x-axis.

Hint: $\int \sqrt{1+x^2}\,dx = \int \sqrt{1+\tan^2\theta}\sec^2\theta\,d\theta = \int \sec\theta\sec^2\theta\,d\theta = \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta - \sec\theta\,d\theta$

(4) Midterm exam

Let $f : [0,1] \to \mathbb{R}$ be a continuous function satisfying

$$\int_0^1 f(x)dx = \frac{1}{2}\,.$$

Show that exist $c \in (0,1)$ such that $f(c) = c$.

(5) Comparion of two method to revolution volume

Given the curve $(x-R)^2+y^2=r^2$ rotating about z-axis, which called torus, find the volume of torus by following method.

(a) [Shell method] $V=\int_{R-r}^{R+r}(2\pi x)y\,dx$

(b) [Disk method] $V=\int_{-r}^{r}\pi x^2\,dy$

Hint:

(a) Let $y=\pm\sqrt{r^2-(x-R)^2}$. The height of the shell is $2\sqrt{r^2-(x-R)^2}$.

(b) Let $x=R\pm\sqrt{r^2-y^2}$. This graph is the large disk minus the small disk. Moreover, let the radius of large disk is $B(y)=R+\sqrt{r^2-y^2}$, and the radius of small disk is $b(y)=R-\sqrt{r^2-y^2}$. Then, the volume $V=\int_{-r}^{r}\pi[B(y)^2-b(y)^2]\,dy$

(6) Given the cycloid curve $\{(x, y) : x = \theta - \sin\theta,\ y = 1 - \cos\theta \quad \text{and} \quad 0 \leq \theta \leq 2\pi\}$, find the following value

(a) the area of between the cycloid curve and x-axis.

(b) the arc-length of the cycloid curve.

(c) the volume of the cycloid curve rotate about x-axis.

(d) the volume of the cycloid curve rotate about y-axis.

(e) the surface area of the cycloid curve rotate bout x-axis.

(f) [*Extra*] Does (a) always hold for 3 times the area?

(g) [*Extra*] Does (b) always hold for 4 times the diameter?

(h) [*Extra*] Could you find the value of $\frac{\sin\theta}{\sqrt{y}}$?

(i) [*Extra*] Is it similar to Snell's law?

(j) [*Extra*] Could you find a path which the particle take minimal time to move between two place by gravity. [**This is my graduate school entrance examination**]