CALCULUS TA SESSION FOR GROUP 1 OCTOBER 7 (VERSION 2)

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(1) Differentiable and Continuous

$$f(x) = \begin{cases} \frac{\sin^2(ax)}{x}, & x > 0 \\ |2x+1| - |2x-1| + b\cos x, & x \leq 0 \end{cases}$$

(a) For what values of a and b will $f(x)$ be continuous at $x = 0$?
(b) For what values of a and b will $f(x)$ be differentiable at $x = 0$?

(2) Differentiable and Continuous *1041 A1 Midterm Problem 1*

Let a function

$$f(x) = \begin{cases} x^\alpha \sin\left(\frac{1}{x^\beta}\right), & x > 0 \\ 0, & x = 0 \\ \frac{\sin\left(x^\beta\right)}{1-\cos x}, & x < 0 \end{cases}$$

(a) For what values of α and β will $f(x)$ be continuous at $x = 0$?
(b) For what values of α and β will $f(x)$ be differentiable at $x = 0$?

Hint: (a) $\alpha > 0$ and $\beta > 0$ (b) $\alpha > 1$ and $\beta > 3$

(3) Find limit

Determine α, β such that

$$\lim_{x\to\infty}\left[\sqrt{4x^2-3x+2}-(\alpha x+\beta)\right]=0$$

(4) Find limit

Compute the following limit

$$\lim_{x\to\infty} x(\sqrt{x^6-3x^5+1}-x^3)\tan\left(\frac{1}{x^3}\right)$$

(5) Inverse Trigonometric Function

Find $\sin \sec^{-1} x$ for $x \leq -1$.

(6) Inverse Trigonometric Function

Find $\sin \sec^{-1} x$.

Hint: $\frac{\sqrt{x^2-1}}{|x|}$. Note that range of the $\cos^{-1}$ is $[0, \pi]$ so $\sin([0, \pi]) \geq 0$.

(7) Find slant asymptotes *1041 A1 Midterm Problem 9*

Let $f(x) = (x^3 + x^2)^{1/3}$. Find all asymptotes of $f(x)$.

Hint: $y = x + \frac{1}{3}$

Remark: If $\lim_{x\to\pm\infty}[f(x) - (mx + b)] = 0$, then there exist asymptotes $L(x) = mx + b$ to the graph $y = f(x)$. Note that if $m = 0$ then $L(x)$ is horizontal asymptote.

Note that if $\lim_{x\to\pm\infty}[f(x) - (mx + b)]$ does't exist, it doesn't mean there is no asymptote. That is, there might be vertical asymptote, *i.e.* $m = \pm\infty$.

Now, there are two step to find the slant asymptotes $L(x) = mx + b$:

0. Determine whether there is vertical (horizontal) asymptotes or not, especially vertical asymptotes.
1. Next, determine whether there is slant asymptotes or not. Consider the following,
$$\lim_{x\to\pm\infty} \frac{f(x) - (mx + b)}{x} = \lim_{x\to\pm\infty} \frac{f(x)}{x} - \left(m + \frac{b}{x}\right).$$
Thus,
$$m = \lim_{x\to\pm\infty} \frac{f(x)}{x}.$$
2. After m has known, it is sufficient to find b.
$$\lim_{x\to\pm\infty}[f(x) - (mx + b)] = 0.$$
Thus,
$$b = \lim_{x\to\pm\infty}[f(x) - mx].$$
Therefore, we find $L(x) = mx+b$ such that $\lim_{x\to\pm\infty}[f(x) - (mx + b)] = 0$, which implies there are asymptotes to graph $y = f(x)$.