

CALCULUS TA SESSION FOR GROUP 1 NOVEMBER 25

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(1) Definition of integral

Find the following limit

$$\lim_{n \rightarrow \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right)$$

(2) Definition of integral

Find the following limit

$$\lim_{n \rightarrow \infty} \left( \sqrt{\frac{1}{n^3}} + \sqrt{\frac{2}{n^3}} + \cdots + \sqrt{\frac{n}{n^3}} \right)$$

(3) Definition of integral

Find the following limit

$$\lim_{n \rightarrow \infty} \left( \frac{3}{\sqrt{n^2 + 3n}} + \frac{3}{\sqrt{n^2 + 6n}} + \cdots + \frac{3}{\sqrt{n^2 + 3n^2}} \right)$$

(4) Definition of integral 1061 A1 Final Problem 6

Find the following limit

$$\lim_{n \rightarrow +\infty} \left( \frac{n}{n^2 + 4 \cdot 1^2} + \frac{n}{n^2 + 4 \cdot 2^2} + \frac{n}{n^2 + 4 \cdot 3^2} + \cdots + \frac{n}{5n^2} \right) = \lim_{n \rightarrow +\infty} \sum_{i=1}^n \frac{n}{n^2 + 4i^2}$$

(5) Definite integral

Find the following integral

$$\int_0^1 x^2 \sqrt[3]{1-x} dx$$

*Hint:* Let  $u = 1 - x$ , so  $du = -dx$  and  $x = 1 - u$

(6) Indefinite integral

Find the following integral

$$\int x^3 \sqrt{x^2 + 1} dx$$

*Hint:* Let  $u = 1 + x^2$ , so  $du = 2x dx$  and  $x^2 = u - 1$

(7) Indefinite integral

Find the following integral

$$\int \frac{\sin^{-1} \sqrt{x}}{\sqrt{x(1-x)}} dx$$

*Hint:* Let  $u = \sqrt{x}$ , so  $2du = \frac{1}{\sqrt{x}}dx$  and  $u^2 = x$ . Let  $v = \sin^{-1} u$ , so  $dv = \frac{1}{\sqrt{1-u^2}}du$ .