CALCULUS TA SESSION FOR GROUP 1 NOVEMBER 25

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(1) Fundamental Theorem of Calculus `1031 A1 Final Problem 1`

Evaluate the following limit,

$$\lim_{x\to 0} \frac{\int_x^{\tan x} \sqrt{1+t^3}dt}{x^3}$$

(2) Fundamental Theorem of Calculus `1091 M Final Problem 1`

Let f be a continuous function on $\mathbb{R}$ such that

$$\int_0^{x^3} f(t)dt = x^3\cos(\pi x)$$

Find $f(1)$.

(3) `1051 A1 Final Problem 1`

Evaluate the integrals

$$\int \frac{1}{\sin x \cos^2 x} dx$$

.

(4) `1061 A1 Final Problem 2`

Evaluate the integrals

$$\int \tan x \log(\cos x) dx$$

.

(5) `1071 A1 Final Problem 3`

Evaluate the integrals

$$\int \frac{x}{\sqrt{25-8x+x^2}}dx$$

.

(6) `1071 A1 Final Problem 3`

Evaluate the integrals

$$\int \frac{e^{2x}}{16-8e^x+e^{2x}}dx$$

.

(7) `1081 M Final Problem 2`
Evaluate the integrals

$$\int \sin^{-1} x dx$$

.

(8) `1081 M Final Problem 1`
Find $f'(2)$ if $f(x) = e^{g(x)}$ and

$$g(x) = \int_4^{x^2} \frac{t}{1+t^4} dt$$

(9) Comparion of two method to revolution volume

Given the curve $(x-R)^2+y^2=r^2$ rotating about z-axis, which called torus, find the volume of torus by following method.

(a) [Shell method] $V=\int_{R-r}^{R+r}(2\pi x)y\,dx$

(b) [Disk method] $V=\int_{-r}^{r}\pi x^2\,dy$

Hint:

(a) Let $y=\pm\sqrt{r^2-(x-R)^2}$. The height of the shell is $2\sqrt{r^2-(x-R)^2}$.

(b) Let $x=R\pm\sqrt{r^2-y^2}$. This graph is the large disk minus the small disk. Moreover, let the radius of large disk is $B(y)=R+\sqrt{r^2-y^2}$, and the radius of small disk is $b(y)=R-\sqrt{r^2-y^2}$. Then, the volume $V=\int_{-r}^{r}\pi[B(y)^2-b(y)^2]\,dy$