CALCULUS TA SESSION FOR GROUP 1 DECEMBER 23

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(1) Compute the following

$$\int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

where μ and σ is a constant.

(2) Consider random variable following the exponential distribution, $X \sim Exp(\lambda)$. The p.d.f is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } o.w. \end{cases}$$

Find the $\mathbb{E}[X]$.

(3) Compute the population mean of Cauchy distribution.

$$\mu = \int_{-\infty}^{\infty} x \frac{1}{\pi} \frac{1}{1+x^2} dx$$

where p.d.f of Cauchy distribution with Cauchy(1) is $\frac{1}{\pi}\frac{1}{1+x^2}$ and the c.d.f is $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x)$.
Remark: Does the weak law of large number fail?

$$\frac{X_1 + \cdots + X_n}{n} = \bar{X} \to \mu$$

in probability. That is,

$$\lim_{n\to\infty} \mathbb{P}\left(\left| \frac{X_1 + \cdots X_n}{n} - 0 \right| > \epsilon \right) = \frac{2}{\pi} \tan^{-1}(\epsilon)$$

Note that limit cannot pass into probability and $\bar{X}$ ~Cauchy(1).

(4) Compute the following

$$\int \frac{x+3}{x^2-x+1}dx$$

(5) Compute the following

$$\int \frac{1}{x^4+2x^2-3}dx$$

(6) Consider the following questions

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{K}\right)P(t)$$

Consider growth rate $r = 1$, and carrying capacity $K = 1$.

(a) Draw the vector field

(b) Identify the equilibrium points (stable point).

(c) Could you use Eular method to find a solution starting from $P(0) = \frac{1}{2}$.

(d) Could you explain this model.