# Calculus I TA Session

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1. **(Differentiable and Continuous)** Modify from 1011 A1 Midterm Problem 2
Suppose that a function
$$f(x) = \begin{cases} x^\alpha \sin(1/x) & \text{if } x > 0 \\ ax + b & \text{if } x \leq 0 \end{cases}$$
where $\alpha \geq 2$.
   (a) if $f$ is continuous at 0, determine the value of $b$.
   (b) if $f$ is differentiable at 0, determine the value of $a$.
   (c) if $f$ is continuously differentiable at 0, determine the value of $\alpha$.

2. **(Inverse differentiation)** 110 (01-05) Midterm Problem 2
Let $f(x)$ be a twice differentiable one-to-one function. Suppose that $f(2) = 1, f'(2) = 3$, $f''(2) = e$. Find the following value
$$\frac{d}{dx} f^{-1}(1) \quad \text{and} \quad \frac{d^2}{dx^2} f^{-1}(1)$$

3. **(Linearization)** 11101 (01-05) Midterm Problem 4
Let $f(x) = x + e^{2(x-1)}$. Let $g(x) = f^{-1}(x)$ be the inverse function of $f(x)$.
   (a) Find $g(2)$ and $g'(2)$.
   (b) Prove that $g''(x) < 0$ for all $x \in \mathbb{R}$.
   (c) Write down the linearization $L(x)$ of $g(x)$ at $x = 2$. Hence determine whether $g(2.1)$ or $L(2.1)$ is larger.

4. **(Extreme value)** 110 (01-05) Midterm Problem 2
Suppose that the equation
$$x^2 \cos(xy) + e^{y^2} - 2x + y = 0$$
is satisfied by a differentiable function $y(x)$ defined on an open interval $I$ containing 1 such that $y(1) = 0$. Besides, we assume that $y''$ exists everywhere on $I$.
   (a) Compute $y'(1)$.
   (b) Compute $y''(1)$.
   (c) Does $y(x)$ attain a local extremum at $x = 1$ ? if your answer is YES, tell the type of local extremum (local maximum or local minimum) and give your reason.

5. **(Linearization)** 110 (01-05) Midterm Problem 4
Consider the function $f(x) = 3x - \tan^{-1}(x - 1)$.
   (a) Show that the equation $3x - \tan^{-1}(x - 1) = 3.01$ has a unique solution.
   (b) Let $g(x)$ be the inverse function of $f$. Find $g(3)$ and $g'(3)$.
   (c) Apply a linear approximation to $g$ to get an estimate of the solution of $f(x) = 3.01$.

6. **(Linearization)** 10701 A1 Midterm Problem 7

   (a) Find the linearization of $f(x) = \sin^{-1} x$ at $x = 0.5$. Denote the linearization by $L(x)$.

   (b) Use linear approximation to estimate $\sin^{-1}(0.49)$.

   (c) Let $g(x) = \sin^{-1} x - L(x)$. Use the Mean Value Theorem twice to estimate $|g(0.49) - g(0.5)|$ and get an upper bound for the quantity.

7. **(Sketch a Graph of a Function)** 1051 B Midterm Problem 7

   Let a function $y = f(x) = \frac{(x+1)^3}{x^2+2x}$

   (a) Find the intervals where $f$ is decreasing or increasing.

   (b) Find the intervals where $f$ is concave up or concave down.

   (c) Find the local maximum and minimum values.

   (d) Find the inflection points.

   (e) Sketch the graph of $y = f(x)$.