

Calculus I TA Session

October, 2023

TA: SING-YUAN YEH

1. **(Implicit differentiation)** 108 A1 Midterm Problem 3 (b)

Suppose function g has the following property

$$g(\sin 3x) = 2(g(x) + x)$$

for any real number x and g is differentiable at $x = 0$. Find $g(0)$ and $g'(0)$.

2. **(Linearization)** 11101 (01-05) Midterm Problem 4

Let $f(x) = x + e^{2(x-1)}$. Let $g(x) = f^{-1}(x)$ be the inverse function of $f(x)$.

- Find $g(2)$ and $g'(2)$.
- Prove that $g''(x) < 0$ for all $x \in \mathbb{R}$.
- Write down the linearization $L(x)$ of $g(x)$ at $x = 2$. Hence determine whether $g(2.1)$ or $L(2.1)$ is larger.

3. **(Extreme value)** 110 (01-05) Midterm Problem 2

Suppose that the equation

$$x^2 \cos(xy) + e^{y^2} - 2x + y = 0$$

is satisfied by a differentiable function $y(x)$ defined on an open interval I containing 1 such that $y(1) = 0$. Besides, we assume that y'' exists everywhere on I .

- Compute $y'(1)$.
- Compute $y''(1)$.
- Does $y(x)$ attain a local extremum at $x = 1$? if your answer is YES, tell the type of local extremum (local maximum or local minimum) and give your reason.