Calculus Ⅱ TA Session

November 30, 2023

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1. **(Improper integral)** 1061 A1 Final Problem 4
(10 points) Find the value of the constant c for which the integral

$$\int_0^\infty \frac{x^2+8}{x^3+8} - \frac{c}{\sqrt{x^2+1}}dx \text{ converges.}$$

Evaluate the integral for this value of c.

2. **(Improper integral)** 1051 A1 Final Problem 3
Determine whether $\int_1^\infty \frac{\tan^{-1}x}{x^2}dx$ converges or diverges. Evaluate the value if it converges.

3. **(Improper integral)** 1111 (01-05) Final Problem 3
Let $f(x)$ be a continuous function on $[1,\infty)$. Note that $f(x)$ is not necessarily non-negative.
   (a) Prove that if $\int_1^\infty |f(t)|\mathrm{d}t$ converges, then $\int_1^\infty f(t)\mathrm{d}t$ also converges. Hint : consider $g(t) = f(t) + |f(t)|$.
   (b) Determine whether $\int_1^\infty \frac{\cos x}{x^2}\ \mathrm{d}x$ is convergent or divergent.
   (c) Determine whether $\int_1^\infty \frac{\sin x}{x}\ \mathrm{d}x$ is convergent or divergent. Hint: Use integration by parts.