Calculus Ⅲ TA Session

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TA: Sing-Yuan Yeh

1. **(Differentiable)** 1102 (01-05) Midterm Problem 1

   Let $f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

   (a) Is $f(x,y)$ continuous at $(0,0)$? Explain.

   (b) Find $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.

   (c) Write down the linearization $L(x,y)$ of $f(x,y)$ at $(0,0)$.

   (d) The function f is differentiable at $(0,0)$ if

   $$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - L(x,y)}{\sqrt{x^2+y^2}} = 0$$

   where $L(x,y)$ is the linearization of $f(x,y)$ at $(0,0)$. Is $f(x,y)$ differentiable at $(0,0)$? Explain.

   (e) Find $f_y(x,y)$ when $(x,y) \neq (0,0)$. Is $f_y(x,y)$ continuous at $(0,0)$? Explain.

2. **(Chain Rule)** 1112 (11-14) Midterm Problem 2

   (14%) The graph $z = f(x,y)$ of the differentiable function f has $2x - 3y + z = 4$ as its tangent plane at the point $(0,0,4)$. The graph $z = g(x,y)$ of the differentiable function g has $x + 2y - z = 3$ as its tangent plane at the point $(0,0,-3)$. Answer the following questions.

   (a) Determine the values: $f(0,0), f_x(0,0), f_y(0,0), g(0,0), g_x(0,0), g_y(0,0)$.

   (b) Use the linearization of f at $(0,0)$ to estimate $f(0.1,-0.1)$.

   (c) Let $h(u,v) = ue^{-2v}$ and $u = f(x,y), v = g(x,y)$. Use the Chain Rule to find the partial derivative

   $$\frac{\partial}{\partial x} h(f(x,y), g(x,y)) \text{ at } x = 0, y = 0$$