Calculus I TA Session (Summer Session)

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1. **IVT**
   Show that if $f : [0,1] \to [0,1]$ is continuous, then there is some c such that $f(c) = c$.

2. **(Definition of Derivatives and Derivatives)** 11001 (13-16) Midterm Problem 2
   Let $f(x) = \begin{cases} |x| \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
   (a) (5%) Determine whether $f(x)$ is continuous at $x = 0$. Explain your answer.
   (b) (5%) Determine whether $f(x)$ is differentiable at $x = 0$. Explain your answer.

3. **(Definition of Continuous and Derivatives)** 10501 A2 Midterm Problem 3
   Suppose that a function
   $$f(x) = \begin{cases} \sin x + b \log(x+1) + c, & \text{if } x \geq 0 \\ e^{x^2}, & \text{if } x < 0 \end{cases}.$$
   1. Find b, c such that f is continuous.
   2. Find b, c such that f is differentiable.

4. **(Definition of Derivatives)** 11001 (01-05) Midterm Problem 3
   Let
   $$f(x) = \begin{cases} x^{\frac{1}{x}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$
   Use the definition of derivatives to compute $f'(0)$ as a limit.

5. **(Definition of Derivatives)** 10801 A Midterm Problem 6
   Let $f(x)$ be a continuous function on $\mathbb{R}$. It is given that
   $$\lim_{h \to 0} \frac{f(h)}{h} = 2020.$$
   Compute $f(0)$. Then, prove that f is differentiable at $x = 0$ and compute $f'(0)$.

6. **(Definition of Derivatives)** 10501 A1 Midterm Problem 5
   Suppose that $f(x)$ is twice differentiable, $\lim_{x \to 1} \frac{(f(x))^3 - 8}{x - 1} = 18$, and $\lim_{t \to 0} \frac{f'(1+t) - f'(1-3t)}{t} = 1$. Find $f(1), f'(1)$ and $f''(1)$.

7. **(Asymptotes)** 10901 (13-17) Midterm Problem 7
   Let
   $$f(x) = \ln \left| \frac{2x+1}{x-1} \right|$$
   Find all vertical asymptotes and horizontal asymptotes of $y = f(x)$.

8. **(Differentiable but not continuously differentiable)**
Let

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Show that f is continuous and f' is not continuous at 0.