Calculus I TA Session (Summer Session)

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1. **(Linearization)** 11101 (01-05) Midterm Problem 4

   Let $f(x) = x + e^{2(x-1)}$. Let $g(x) = f^{-1}(x)$ be the inverse function of $f(x)$.

   (a) Find $g(2)$ and $g'(2)$.

   (b) Prove that $g''(x) < 0$ for all $x \in \mathbb{R}$.

   (c) Write down the linearization $L(x)$ of $g(x)$ at $x = 2$. Hence determine whether $g(2.1)$ or $L(2.1)$ is larger.

2. **(Extreme value)** 110 (01-05) Midterm Problem 2

   Suppose that the equation

   $$x^2 \cos(xy) + e^{y^2} - 2x + y = 0$$

   is satisfied by a differentiable function $y(x)$ defined on an open interval I containing 1 such that $y(1) = 0$. Besides, we assume that y'' exists everywhere on I.

   (a) Compute $y'(1)$.

   (b) Compute $y''(1)$.

   (c) Does $y(x)$ attain a local extremum at $x = 1$? if your answer is YES, tell the type of local extremum (local maximum or local minimum) and give your reason.

3. **(Linearization)** 110 (01-05) Midterm Problem 4

   Consider the function $f(x) = 3x - \tan^{-1}(x - 1)$.

   1. Show that the equation $3x - \tan^{-1}(x - 1) = 3.01$ has a unique solution.
   2. Let $g(x)$ be the inverse function of f. Find $g(3)$ and $g'(3)$.
   3. Apply a linear approximation to g to get an estimate of the solution of $f(x) = 3.01$.

4. **(Linearization)** 10701 A1 Midterm Problem 7

   1. Find the linearization of $f(x) = \sin^{-1} x$ at $x = 0.5$. Denote the linearization by $L(x)$.
   2. Use linear approximation to estimate $\sin^{-1}(0.49)$.
   3. Let $g(x) = \sin^{-1} x - L(x)$. Use the Mean Value Theorem twice to estimate $|g(0.49) - g(0.5)|$ and get an upper bound for the quantity.